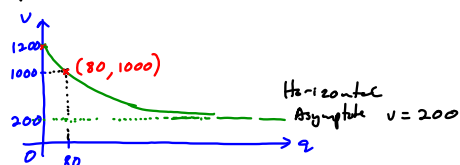


Ex 1.5 Page 31

⑦ The daily variable cost v of

Solution:-

$$\text{we want } v(q) = \frac{aq+b}{q+c}$$

When $q=0$, then $v = \$1,200$ When $q=80$, then $v = \$1,000$ And as $q \rightarrow \infty$, then $v \rightarrow \$200$ The graph of $v(q)$ is① When $q=0$, $v=1200$

$$\text{so } 1200 = \frac{a(0)+b}{0+c}$$

$$\Rightarrow \frac{b}{c} = 1200$$

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$$\Rightarrow b = 1200c \quad - (1)$$

② The function passes (80, 1000)
When $q=80$, then $v=1000$

$$1000 = \frac{a(80)+b}{80+c}$$

$$\Rightarrow 1000(80+c) = 80a+b$$

$$\Rightarrow 80000 + 1000c = 80a+b \quad - (2)$$

③ As $q \rightarrow \infty$, then $v \rightarrow 200$ $\therefore v=200$ is a non-vertical asymptote

$$\text{Note: } v = \frac{aq+b}{q+c}$$

can be rewritten as

$$v = a + \frac{b-aq}{q+c}$$

$$\Rightarrow v = a + \frac{b-ac}{q+c} \quad \therefore \frac{aq+b}{q+c} = a + \frac{b-ac}{q+c}$$

 $v=a$ is the non-vertical asymptoteSince $v=200$ is the non-vertical asymptote

$$\Rightarrow a = 200 \quad - (3)$$

Proof:-

$$\begin{array}{r|rr} -c & a & b \\ \hline & 0 & -ac \\ & a & b-ac \end{array}$$

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So far we have

$$b = 1200c \quad - (1)$$

$$80000 + 1000c = 80a + b \quad - (2)$$

$$a = 200 \quad - (3)$$

Solving all the above,

$$a = 200, b = 384000 \text{ and } c = 320$$

$$\therefore v(q) = \frac{200q + 384000}{q + 320}; \quad q \geq 0$$

Howie solve it.

$$b = 1200c \quad - (1) \text{ and } a = 200 \quad - (3)$$

plug into

$$80000 + 1000c = 80a + b \quad - (2)$$

$$\Rightarrow 80000 + 1000c = 80(200) + 1200c$$

$$\Rightarrow 80000 + 1000c = 16000 + 1200c$$

$$\Rightarrow 1000c - 1200c = 16000 - 80000$$

$$\Rightarrow -200c = -64000$$

$$\Rightarrow c = \frac{-64000}{-200} = 320$$

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And plug this into
 $b = 1200c$
 $\Rightarrow b = (1200)(320) = 384000$
 $\therefore a = 200, b = 384000 \text{ and } c = 320$

(10) Using the daily.....

Solution:- we found that $v = \frac{200q + 384000}{q + 320}$
 and given $F = \$1,300$

we know that the total costs

$$\begin{aligned} C &= F + vq \\ \Rightarrow C &= 1300 + \left(\frac{200q + 384000}{q + 320} \right)(q) \\ &= \frac{1300(q + 320)}{q + 320} + \frac{200q^2 + 384000q}{q + 320} \\ &= \frac{1300q + (1300)(320) + 200q^2 + 384000q}{q + 320} \\ \Rightarrow C(q) &= \frac{200q^2 + 385300q + 416000}{q + 320}; q \geq 0 \end{aligned}$$

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(11) The mining company from.....

Solution:-

we want the revenue,

$$R = pq$$

given $p = \$350$

so $R = 350q$

Next, we want the profit

$$P = R - C$$

where $R = 350q$ and $C = \frac{200q^2 + 385300q + 416000}{q + 320}$

so $P = 350q - \frac{200q^2 + 385300q + 416000}{q + 320}$

$$\begin{aligned} \Rightarrow P &= \frac{350q(q + 320)}{q + 320} - \frac{200q^2 + 385300q + 416000}{q + 320} \\ &= \frac{350q^2 + (350q)(320) - (200q^2 + 385300q + 416000)}{q + 320} \\ &= \frac{350q^2 + 112000q - 200q^2 - 385300q - 416000}{q + 320} \end{aligned}$$

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$$\Rightarrow P = \frac{150q^2 - 273000q - 416000}{q + 320}; q \geq 0$$

Break-even occurs when $P = 0$,

$$\Rightarrow \frac{150q^2 - 273000q - 416000}{q + 320} = 0$$

$$\Rightarrow 150q^2 - 273000q - 416000 = 0$$

$$\Rightarrow q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{array}{l} \text{Here, } a = 150 \\ b = -273000 \\ c = -416000 \end{array}$$

$$\Rightarrow q = \frac{-(-273000) \pm \sqrt{(-273000)^2 - 4(150)(-416000)}}{2(150)}$$

$$\Rightarrow q = \frac{273000 \pm \sqrt{74778600000}}{300}$$

$$= \frac{273000 \pm 273456.761}{300}$$

$$\Rightarrow q = \frac{273000 - 273456.761}{300} \text{ or } q = \frac{273000 + 273456.761}{300}$$

But since $q \geq 0$, we reject the -ve answer

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and accept

$$2 \approx \frac{273000 + 273456.761}{300}$$

$$\Rightarrow 2 \approx \frac{546456.761}{300}$$

$$\Rightarrow 2 \approx 1822 \quad \leftarrow \text{This is the approximate break-even point.}$$

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1.6 Function Composition

= Function within a function

If $f(x)$ and $g(x)$ are functions, we can compose them as follows

$$f(g(x))$$

$g(x)$ is the argument of $f(x)$.

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① Given $f(x) = 3x - 6$ and $g(x) = x^2$

(i) $f(g(x)) = ???$

Given $f(x) = 3x - 6$

$\Rightarrow f(g) = 3g - 6$ Everywhere x occurs, must be changed to g .

$\Rightarrow f(g(x)) = 3g(x) - 6$ More precisely, g is in terms of x .

$\Rightarrow f(g(x)) = 3x^2 - 6$ Better way of writing it.

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(ii) $g(f(x)) = ???$

Given $g(x) = x^2$

$\Rightarrow g(f) = f^2$

$\Rightarrow g(f(x)) = (f(x))^2$

$\Rightarrow g(f(x)) = (3x - 6)^2$ since $f(x) = 3x - 6$

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$f(\text{Chelsea}) =$

Dear Chelsea,

How I love thee, Chelsea!

I will climb the highest mountain for you, Chelsea.

I will swim the widest ocean for you, Chelsea.

Chelsea, Chelsea, Chelsea, you are the sunshine of my apple.

Love,
John

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④ Given $f(x) = \sqrt{2x - 11}$ and $g(x) = \frac{x}{x - 11}$

Domain of f : $2x - 11 \geq 0$

$\Rightarrow x \geq \frac{11}{2} \Rightarrow x \geq 5.5$

Domain of g : $x - 11 \neq 0$

$\Rightarrow x \neq 11$

(i) $f(g(x)) = ???$

Start with $f(x) = \sqrt{2x - 11}$

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$$\begin{aligned} \Rightarrow f(g) &= \sqrt{2g-11} \\ \Rightarrow f(g(x)) &= \sqrt{2g(x)-11} \\ \Rightarrow f(g(x)) &= \sqrt{2\left(\frac{x}{x-11}\right)-11} \\ &= \sqrt{\frac{2x}{x-11}-11} = \sqrt{\frac{2x}{x-11}-\frac{11(x-11)}{x-11}} \\ &= \sqrt{\frac{2x-11(x-11)}{x-11}} = \sqrt{\frac{2x-11x+121}{x-11}} \end{aligned}$$

Remember $g(x)$ is not x multiply x

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$$\Rightarrow f(g(x)) = \sqrt{\frac{121-9x}{x-11}}$$

To find the domain, we need

$$\frac{121-9x}{x-11} \geq 0 \text{ and } x-11 \neq 0$$

Let's do a sign analysis.

Assume $(121-9x)(x-11) = 0$

$$\Rightarrow 121-9x = 0 \text{ or } x-11 = 0$$

$$\Rightarrow x = \frac{121}{9} \text{ or } x = 11$$

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	$5.5 < x < 11$	$11 < x < \frac{121}{9}$	$x > \frac{121}{9}$
Sign of $121-9x$	+ve	+ve	-ve
Sign of $x-11$	-ve	+ve	+ve
Sign of $\frac{121-9x}{x-11}$	-ve	+ve	-ve

$\therefore$ The interval on which $\frac{121-9x}{x-11} \geq 0$ and $x \neq 11$

is $11 < x \leq \frac{121}{9}$

or $x \in \left(11, \frac{121}{9}\right]$

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Function decomposition

① Given $f(g(x)) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$

Let $g(x) = \sqrt{x}$, so we have

$$f(g(x)) = \frac{g(x)+1}{g(x)-1}$$

$$\Rightarrow f(x) = \frac{x+1}{x-1}$$

Replacing $g(x)$ with x

$\therefore f(x) = \frac{x+1}{x-1}$ and $g(x) = \sqrt{x}$

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Function decomposition is not unique.

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